Mechanisms for the radical graft polymerization of vinyl and/or acrylic monomers on cellulose*

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A model for the heterogeneous graft polymerization of vinyl and/or acrylic monomers on cellulose initiated by ceric ions is developed. The model can be easily extended to other (poly)hydroxylic substrates, other one-electron oxidants and homogeneous systems. Three component mechanisms are suggested to be relevant in a radical graft polymerization: *from, onto* and *from-to*. Several competitive reactions appear in the polymerization scheme that lead to related mechanistic cases. The electronic nature of the monomer and the number of polymerization sites are taken into account. Expressions for reaction parameters as a function of reaction variables are deduced. Some structural knowledge of the copolymer is derived from the model.

(Keywords: graft polymerization; heterogeneous polymerization; radical polymerization)

INTRODUCTION

Grafting is a useful method to modify some properties of natural and synthetic polymers. The ceric-ion method is one of the most interesting methods to graft olefins on polymers presenting oxidizable functional groups. Free radicals are formed on a trunk polymer by ceric ions.

In a previous paper[1], effort was made to gain some insight into the complex heterogeneous polymerization of vinyl and/or acrylic monomers on cellulose. Some mechanistic considerations were made on the graft polymerization and homopolymerization reactions by taking into account the chemical nature of olefins and cellulose. The possible identity of the copolymerization and homopolymerization sites is also of prime importance from a mechanistic point of view. Expressions for the reaction parameters were developed by assuming that both polymerization sites were different. This behaviour was assumed on the basis of a discriminating effect of the fibrous morphology of the insoluble cotton.

Results obtained from the copolymerization of vinyl acetate–methyl acrylate mixtures on cellulose[2] evidencing the identity of the copolymerization and homopolymerization sites prompted us to take a more comprehensive view of the polymerization scheme. A copolymerization model, with its component mechanisms, is developed by taking into account the heterogeneous nature of the graft polymerization, the electronic character of the monomer and the number of polymerization sites.

GRAFTING MECHANISMS *FROM*, *ONTO* AND *FROM-TO*

Several chemical processes can occur when vinyl and/or acrylic monomers are grafted on cellulose after initiation with ceric ions in a heterogeneous aqueous medium. It is suggested that the present model can be easily adapted, if necessary, to other oxidizable compounds (e.g. alcohols, amines, starch, poly(vinyl alcohol), polyamides), to other one-electron oxidants (e.g. Co(III), V(V), Mn(III)) and to homogeneous (co)polymerizations, in cases where chemical systems analogous to or simpler than cellulose and ceric ions in a heterogeneous medium are taken into account. The chemical processes are shown in the following polymerization scheme.

Initiation

$$\mathrm{S} + \mathrm{Ce}^{\mathrm{IV}} \underset{\alpha_{-1}}{\overset{\alpha_1}{\rightleftharpoons}} \mathrm{S|Ce}^{\mathrm{IV}} \overset{K_d}{\rightleftharpoons} \{\mathrm{S{-}Ce}^{\mathrm{IV}}\} \xrightarrow{k_d} \mathrm{S}^{\bullet} + \mathrm{Ce}^{\mathrm{III}} + \mathrm{H}^{+} \tag{1}$$

$$\mathrm{S}^{\bullet} + \mathrm{M} \underset{\alpha_{-2}}{\overset{\alpha_2}{\rightleftharpoons}} \mathrm{M|S}^{\bullet} \overset{K_i}{\rightleftharpoons} \{\mathrm{M{-}S}^{\bullet}\} \xrightarrow{k_i} \mathrm{S{-}M}^{\bullet} \tag{2}$$

$$\mathrm{M} + \mathrm{Ce}^{\mathrm{IV}} \underset{\alpha_{-3}}{\overset{\alpha_3}{\rightleftharpoons}} \mathrm{M|Ce}^{\mathrm{IV}} \overset{K_i'}{\rightleftharpoons} \{\mathrm{M{-}Ce}^{\mathrm{IV}}\} \xrightarrow{k_i'} \mathrm{M}^{\bullet} + \mathrm{Ce}^{\mathrm{III}} + \mathrm{H}^{+} \tag{3}$$

Propagation

$$\mathrm{S{-}M}_n^{\bullet} + \mathrm{M} \xrightarrow{k_p} \mathrm{S{-}M}_{n+1}^{\bullet} \tag{4}$$

$$\mathrm{M}_n^{\bullet} + \mathrm{M} \xrightarrow{k_p'} \mathrm{M}_{n+1}^{\bullet} \tag{5}$$

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* Dedicated to the memory of Professor Miguel Valero

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Termination

$$\mathrm{S{-}M}_n^{\bullet}+\mathrm{Ce^{IV}}\underset{\alpha_{-4}}{\overset{\alpha_4}{\rightleftharpoons}}\mathrm{Ce^{IV}|S{-}M}_n^{\bullet}\overset{K_{\mathrm{t}}}{\rightleftharpoons}\{\mathrm{Ce^{IV}{-}S{-}M}_n^{\bullet}\}\xrightarrow{k_{\mathrm{t1}}}\mathrm{graft}+\mathrm{Ce^{III}}+\mathrm{H^+} \tag{6}$$

$$\mathrm{S{-}M}_n^{\bullet}+\mathrm{S{-}M}_n^{\bullet}\xrightarrow{k_{\mathrm{t2}}}\mathrm{graft} \tag{7}$$

$$\mathrm{S{-}M}_n^{\bullet}+\mathrm{M}_n^{\bullet}\underset{\alpha_{-7}}{\overset{\alpha_7}{\rightleftharpoons}}\mathrm{S{-}M}_n^{\bullet}|\mathrm{M}_n^{\bullet}\xrightarrow{k_{\mathrm{t2c}}}\mathrm{graft} \tag{8}$$

$$\mathrm{S{-}M}_n^{\bullet}+\mathrm{M}_n^{\bullet}\underset{\alpha_{-7}}{\overset{\alpha_7}{\rightleftharpoons}}\mathrm{S{-}M}_n^{\bullet}|\mathrm{M}_n^{\bullet}\xrightarrow{k_{\mathrm{t2d}}}\mathrm{graft}+\mathrm{homopolymer} \tag{9}$$

$$\mathrm{S{-}M}_n^{\bullet}+\mathrm{S}^{\bullet}\underset{\alpha_{-6}}{\overset{\alpha_6}{\rightleftharpoons}}\mathrm{S{-}M}_n^{\bullet}|\mathrm{S}^{\bullet}\xrightarrow{k_{\mathrm{tR}}}\mathrm{graft} \tag{10}$$

$$\mathrm{M}_n^{\bullet}+\mathrm{S}^{\bullet}\underset{\alpha_{-5}}{\overset{\alpha_5}{\rightleftharpoons}}\mathrm{M}_n^{\bullet}|\mathrm{S}^{\bullet}\xrightarrow{k_{\mathrm{tHc}}}\mathrm{graft} \tag{11}$$

$$\mathrm{M}_n^{\bullet}+\mathrm{S}^{\bullet}\underset{\alpha_{-5}}{\overset{\alpha_5}{\rightleftharpoons}}\mathrm{M}_n^{\bullet}|\mathrm{S}^{\bullet}\xrightarrow{k_{\mathrm{tHd}}}\mathrm{homopolymer}+\mathrm{cellulosic\ product} \tag{12}$$

$$\mathrm{M}_n^{\bullet}+\mathrm{Ce^{IV}}\underset{\alpha_{-4}}{\overset{\alpha_4}{\rightleftharpoons}}\mathrm{Ce^{IV}|M}_n^{\bullet}\overset{K_{\mathrm{t}}}{\rightleftharpoons}\{\mathrm{Ce^{IV}{-}M}_n^{\bullet}\}\xrightarrow{k'_{\mathrm{t1}}}\mathrm{homopolymer}+\mathrm{Ce^{III}}+\mathrm{H^+} \tag{13}$$

$$\mathrm{M}_n^{\bullet}+\mathrm{M}_n^{\bullet}\xrightarrow{k'_{\mathrm{t2}}}\mathrm{homopolymer} \tag{14}$$

$$\mathrm{S}^{\bullet}+\mathrm{Ce^{IV}}\underset{\alpha_{-1}}{\overset{\alpha_1}{\rightleftharpoons}}\mathrm{Ce^{IV}|S}^{\bullet}\overset{K_0}{\rightleftharpoons}\{\mathrm{Ce^{IV}{-}S}^{\bullet}\}\xrightarrow{k_0}\mathrm{oxidation\ products}+\mathrm{Ce^{III}}+\mathrm{H^+} \tag{15}$$

Here S is the cellulosic substrate; M is the monomer; α are diffusion constants of species between their own medium and a state of encounter suitable for the species interaction; K represents the formation constant of two species to constitute an adduct; k represents the rate constant of a reaction step; k_{t2}, k'_{t2} and k_{tR} are rate constants of mutual termination of free radicals; k_{t2c}, k_{tHc} and k_{t2d}, k_{tHd} are termination rate constants by combination and disproportionation of radical chains, respectively. The diffusion constants α and the diffusion ratio of direct and reverse diffusion constants, X, are measures of the accessibility of the reactants to the state of encounter and are used, in the same way as in a previous work[1], to take into account the usual heterogeneous nature of the reaction medium.

Cellulosic radicals $\mathrm{S}^{\bullet}$ formed according to equation (1) can be trapped by several species: by M (equation (2)) leading to grafting initiation; by radicals $\mathrm{S{-}M}_n^{\bullet}$ (equation (10)) and $\mathrm{M}_n^{\bullet}$ (equation (11)) leading to graft polymer; by $\mathrm{M}_n^{\bullet}$ (equation (12)) leading to homopolymer plus a cellulosic residue; and by $\mathrm{Ce^{IV}}$ (equation (15)) yielding oxidized cellulose. The radicals $\mathrm{S}^{\bullet}$ trapped by M yield radical chains $\mathrm{S{-}M}_n^{\bullet}$ by adding M through the propagation reaction (equation (4)), and then they undergo mutual termination (equation (7)), linear termination by $\mathrm{Ce^{IV}}$ (equation (6)), and mutual termination with radical chains $\mathrm{M}_n^{\bullet}$ (equations (8) and (9)) and with cellulosic radicals (equation (10)). If termination with radicals $\mathrm{M}_n^{\bullet}$ is by combination (equation (8)) the resulting product is graft polymer, and if termination is by disproportionation (equation (9)) $\mathrm{S{-}M}_n^{\bullet}$ leads to graft polymer and $\mathrm{M}_n^{\bullet}$ leads to homopolymer.

The monomer radicals $\mathrm{M}^{\bullet}$ formed in equation (3) added monomer according to the propagation reaction (equation (5)) giving rise to radical chains $\mathrm{M}_n^{\bullet}$. These polymer radicals terminate by ceric ions (equation (13)) or mutually (equation (14)) to yield homopolymer. They can also terminate with radicals $\mathrm{S{-}M}_n^{\bullet}$ and $\mathrm{S}^{\bullet}$ by combination (equations (8) and (11)) to yield graft polymer or by disproportionation (equations (9) and (12)) to yield graft polymer and homopolymer plus a cellulosic residue, respectively.

Grafting on cellulose can arise from: (a) addition of monomer onto cellulosic radicals followed by propagation of the grafted radical chains (mechanism *from* of grafting); (b) combination reaction of radical homopolymer chains with cellulosic radicals (mechanism *onto* of grafting); and (c) combination reaction of radical homopolymer chains with grafted radical chains (mechanism *from-to* of grafting).

These three mechanisms contribute, to greater or lesser extent, to the total graft polymer. Each mechanism of grafting requires the assistance of some characteristics of the reactants: high reactivity of $\mathrm{S}^{\bullet}$ with M (mechanism *from*); high reactivity of $\mathrm{Ce^{IV}}$ with M and of $\mathrm{S}^{\bullet}$ with $\mathrm{M}_n^{\bullet}$ (mechanism *onto*); and high reactivity of $\mathrm{S}^{\bullet}$ with M, $\mathrm{Ce^{IV}}$ with M, and $\mathrm{S{-}M}_n^{\bullet}$ with $\mathrm{M}_n^{\bullet}$ (mechanism *from-to*). In some way the mechanism *from-to* is a hybrid of the *from* and *onto* mechanisms. Mechanism *from-to* complements mechanism *from* to constitute the method *from* of grafting, and mechanism *onto* constitutes by itself the method *onto* of grafting. The presence of any of these mechanisms demands the mutual accessibility of the reactive species. This could take special significance when the mechanisms *onto* or *from-to* occur in the usual heterogeneous polymerization, as the species are bulky polymer radicals. The morphology of insoluble cellulose, presenting channels and pores, could lead to different graft polymerization and homopolymerization sites if the mutual accessibility of radicals $\mathrm{S{-}M}_n^{\bullet}$ and $\mathrm{M}_n^{\bullet}$ becomes hindered.

As discussed elsewhere[1], the mechanism *from* of grafting on cellulose (or other oxidizable substrates yielding radicals with a high-energy SOMO*) should be favoured for electron-poor olefin monomers (those with electron-withdrawing substituents) and the grafting *onto* mechanism should be favoured for electron-rich olefins (those with electron-donating substituents). On the other hand, the mechanism *from-to* could acquire relevance at high monomer concentrations (as will be shown later), for olefins of intermediate electronic characteristics and for monomer mixtures in which low-energy HOMO* and high-energy LUMO* olefins are present.

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* HOMO = highest occupied molecular orbital
LUMO = lowest unoccupied molecular orbital
SOMO = singly occupied molecular orbital

COMPETITIVE REACTIONS, APPROXIMATIONS AND CASES

Estimation of reaction parameters

The following expressions for reaction parameters as a function of reaction variables are developed by adopting the same general criteria of a previous work[1]. At present, a more comprehensive model is developed to take into account, as a whole, the three mechanisms of grafting (*from*, *onto* and *from-to*). The mechanisms *from* and *onto* have been previously[3] suggested to account for results obtained when grafting monomers on natural and synthetic polymers. The mechanism *from-to* is expected to occur in systems where active radicals on a trunk polymer and on a monomer are formed simultaneously, giving rise to the corresponding polymer radicals by successive addition of monomer, and the mutual accessibility of both polymer radicals for termination by combination is not prevented. Some results from polymerizations of vinyl and acrylic monomers are accounted for by the assistance of this mechanism[4,5].

Some reaction parameters are derived from the polymerization scheme: R_g is the grafting reaction rate, R_h is the rate of homopolymer formation, R_p is the rate of total polymer formation, R_{Ce} is the rate of consumption of ceric ions, *GE* (grafting efficiency) is the fraction of the total polymer formed that is grafted on cellulose, v_g is the chain length of the grafted polymer, v_h is the chain length of the homopolymer, N_g is the (mole) number of grafted polymer chains, N_h is the (mole) number of homopolymer chains, *GF* (grafting frequency) is the number of grafts per cellulose chain, and *BE* (branching efficiency) is the fraction of cellulose radicals reacting either with M (mechanisms *from* and *from-to*) or $\mathrm{M}_n^{\bullet}$ (mechanism *onto*) leading to grafting.

Three components, R_{g1}, R_{g2} and R_{g3}, contribute to the total grafting rate, R_g, corresponding to the grafting mechanisms *from*, *from-to* and *onto*, respectively:

$$R_g = R_{g1} + R_{g2} + R_{g3} = k_p[\mathrm{M}][\mathrm{S{-}M}_n^{\bullet}] + f_1 k_p'[\mathrm{M}][\mathrm{M}_n^{\bullet}] + f_2 k_p'[\mathrm{M}][\mathrm{M}_n^{\bullet}] \tag{16}$$

where f_1 and f_2 are combination efficiencies, i.e. the fractions of radical chains $\mathrm{M}_n^{\bullet}$ terminating by combination with radicals $\mathrm{S{-}M}_n^{\bullet}$ and $\mathrm{S}^{\bullet}$, respectively. Accordingly, the homopolymerization rate R_h is the total homopolymerization rate R_H minus the rate of formation of homopolymer radicals that will become anchored to cellulose through a combination reaction:

$$R_h = R_H - R_{g2} - R_{g3} = k_p'[\mathrm{M}][\mathrm{M}_n^{\bullet}] - f_1 k_p'[\mathrm{M}][\mathrm{M}_n^{\bullet}] - f_2 k_p'[\mathrm{M}][\mathrm{M}_n^{\bullet}] \tag{17}$$

R_{g2} and R_{g3} are negligible when the reaction conditions prompt the termination of radical chains essentially by ceric ions.

Applying steady-state conditions to each concentration $[\mathrm{S}^{\bullet}]$, $[\mathrm{S{-}M}_n^{\bullet}]$ and $[\mathrm{M}_n^{\bullet}]$, there are several competitive reactions consuming each radical species:

for $[\mathrm{S}^{\bullet}]$

$$R_d = R_i + R_0 + R_{tH} + R_{tR} \tag{18}$$

for $[\mathrm{S{-}M}_n^{\bullet}]$

$$R_i = R_T = R_{t2} + R_t + R_{t1} + R_{tR} \tag{19}$$

for $[\mathrm{M}_n^{\bullet}]$

$$R_i' = R_T' = R_{t2}' + R_t + R_{t1}' + R_{tH} \tag{20}$$

Here R means rate of reaction in the polymerization scheme: R_d, R_i, R_0, R_{tH}, R_{tR}, R_{t2}, R_t, R_{t1}, R_i', R_{t2}' and R_{t1}' for equations (1), (2), (15) (11)/(12), (10), (7), (8)/(9), (6), (3), (14) and (13), respectively; and R_T and R_T' are the total rates of termination of radicals $\mathrm{S{-}M}_n^{\bullet}$ and $\mathrm{M}_n^{\bullet}$, respectively. According to the meaning of the formation constants K, the formation of adducts suggested for some of the component reactions in the polymer scheme could take the corresponding reaction rates towards constancy when one of the two species that give rise to them is present in a high enough concentration.

In order to find suitable expressions for each radical concentration to be used in developing expressions for reaction parameters, adequate *approximations* by neglecting some competitive reactions are made: approximations (A), (B), (C) and (D) mean R_d equals R_i, R_0, R_{tH} and R_{tR}, respectively; approximations (I) and (II) mean R_d equals $R_0 + R_i$ (where $R_0 \gg R_i$) and $R_0 + R_{tH}$ (where $R_0 \gg R_{tH}$), respectively; approximations (a), (b), (c) and (d) mean R_T equals R_{t2}, R_t, R_{t1} and R_{tR}, respectively; approximations (α), (β) and (γ) mean R_T' equals $R_t + R_{t2}'$, $R_t + R_{t1}'$ and $R_t + R_{tH}$, respectively; and approximations (α') and (β') mean R_T' equals $R_{tH} + R_{t2}'$ and $R_{tH} + R_{t1}'$, respectively.

Several mechanistic *cases* appear by assembling adequate approximations from the three steady-state conditions. Consequently, each case results from the predominance of some competitive reactions over others. Every case means a mechanistic and structural picture of graft polymer and homopolymer formation. As an explanatory example, the case (BIbα) means that most of the cellulose radicals formed are oxidized by ceric ions (approximation (B)) and the remainder of these radicals are trapped by monomer (approximation (I)), the radicals $\mathrm{S{-}M}_n^{\bullet}$ formed in this fashion terminate mutually with radicals $\mathrm{M}_n^{\bullet}$ (approximation (b)), and the remainder of the radicals $\mathrm{M}_n^{\bullet}$ self-terminate mutually (approximation (α)). As this case comprises the approximation (b) (mechanism *from-to* occurs) the mutual accessibility of the radicals $\mathrm{S{-}M}_n^{\bullet}$ and $\mathrm{M}_n^{\bullet}$ must be ensured. The mechanistic behaviour of the case (BIbα) allows the formation of copolymer constituted by some branches of synthetic polymer linked to 'highly' oxidized cellulose and an amount of homopolymer.

The approximations made and, as a consequence, the resultant mechanistic cases are inherently related to the three components of the grafting rate, R_{g1}, R_{g2} and R_{g3}, or, what is the same, to the three grafting mechanisms *from*, *from-to* and *onto*, respectively.

In the cases where the approximation (A) is made, most of the radicals $\mathrm{S}^{\bullet}$ are trapped by monomer leading to radicals $\mathrm{S{-}M}_n^{\bullet}$, and only a minor amount of radicals $\mathrm{S}^{\bullet}$ remains available to terminate by combination with radicals $\mathrm{M}_n^{\bullet}$. Therefore, the contribution of R_{g3} to R_g is negligible. Only when the approximation (b) is also made does R_{g2} contribute to R_g because radicals $\mathrm{S{-}M}_n^{\bullet}$ terminate by combination with radicals $\mathrm{M}_n^{\bullet}$. In this way, $R_g = R_{g1}$ in the cases (Aa), (Ac) and (Ad); $R_g = R_{g1} + R_{g2}$ in the cases (Abα) and (Abβ); and $R_g = R_{g1} + R_{g2} + R_{g3}$ (though R_{g3} is quantitatively much less important) in the case (Abγ).

In the cases where the approximation (B) is made, most of the radicals $S^\bullet$ are oxidized by ceric ions. The remainder of the radicals $S^\bullet$, though a small number when compared with those oxidized, can constitute an appreciable amount that reacts (i) with monomer (approximation (I)) giving rise to radicals $S\text{–}M_n^\bullet$ and therefore to contributions to R_g of R_{g1} and R_{g2} (the latter only when the approximation (b) is also made), or (ii) with radicals $M_n^\bullet$ (approximation (II)) leading to a contribution of R_{g3} to R_g. So, $R_g = R_{g1}$ in the cases (BIa), (BIc) and (BId); $R_g = R_{g1} + R_{g2}$ in the cases (BIbα) and (BIbβ); $R_g = R_{g1} + R_{g2} + R_{g3}$ (though R_{g3} is quantitatively much less important) in the case (BIbγ); and $R_g = R_{g3}$ in the cases (BIIα′) and (BIIβ′).

In the cases where the approximation (C) is made, most of the radicals $S^\bullet$ are terminated by combination with radicals $M_n^\bullet$ leading to a contribution of R_{g3} to R_g. The contribution of R_{g1} (and consequently of R_{g2}) is negligible, as only a minor amount of the radicals $S^\bullet$ reacts with monomer yielding radicals $S\text{–}M_n^\bullet$. Therefore, $R_g = R_{g3}$ in the cases (Cα′) and (Cβ′).

At last, in the cases where the approximation (D) is made, a portion of the radicals $S^\bullet$ is trapped by monomer leading to radicals $S\text{–}M_n^\bullet$, which terminate mutually with another portion of the radicals $S^\bullet$. Therefore, $R_g = R_{g1}$ in the case (Dd). The cases (Ad), (BId) and (Dd) show the same expressions for R_g. Nevertheless, there is no internal coherence in each one of these cases. For instance, the case (Ad) means that *most* of the radicals $S^\bullet$ add monomer giving rise to radicals $S\text{–}M_n^\bullet$ (approximation (A)) and they terminate mutually with a residual, *minor* portion of radicals $S^\bullet$. This is incoherent, as each radical $S\text{–}M_n^\bullet$ demands another radical $S^\bullet$ for mutual termination. A case, (Yd), is taken as representative of the three cases and the parameter expressions are developed by assuming that identical portions of radicals $S^\bullet$ add monomer and terminate with radicals $S\text{–}M_n^\bullet$.

Expressions for the reaction parameters

The parameter expressions are developed assuming reactivity-controlled reactions in the polymerization scheme. When any reaction in the polymerization scheme is diffusion-controlled, new expressions can be derived by substituting the product kKX by the corresponding diffusion constant α of the same reaction[1]. For instance, when the confluence of ceric ion and cellulose to their state of encounter (equation (1)) is diffusion-controlled, the product $k_d K_d X_1$ is substituted by α_1 in the corresponding expressions of the reaction parameters. The variables x and n that appear in some of the following equations are explained after equation (106).

Case (Aa)

$$R_g = k_p[M](k_d K_d X_1[S][Ce^{IV}]/2k_{t2})^{0.5} \qquad (21)$$

$$R_h = k'_p[M]^{1.5}(k'_i K'_i X_3[Ce^{IV}]/2k'_{t2})^{0.5} \qquad (22)$$

$$GE = \frac{k_p(2k'_{t2}k_d K_d X_1[S])^{0.5}}{k_p(2k'_{t2}k_d K_d X_1[S])^{0.5} + k'_p(2k'_{t2}k'_i K'_i X_3[M])^{0.5}} \qquad (23)$$

$$R_{Ce} = [Ce^{IV}](k_d K_d X_1[S] + k'_i K'_i X_3[M]) \qquad (24)$$

$$v_g = k_p[M]/(2k_{t2}k_d K_d X_1[S][Ce^{IV}])^{0.5} \qquad (25)$$

$$v_h = k'_p[M]^{0.5}/(2k'_{t2}k'_i K'_i X_3[Ce^{IV}])^{0.5} \qquad (26)$$

$$N_g = nk_d K_d X_1[S][Ce^{IV}] \qquad (27)$$

$$N_h = nk'_i K'_i X_3[Ce^{IV}][M] \qquad (28)$$

$$GF \propto N_g = nk_d K_d X_1[S][Ce^{IV}] \qquad (27)$$

$$BE = 1 \qquad (29)$$

Case (Abα)

$$R_g = \frac{k_d K_d X_1[S][Ce^{IV}]^{0.5}}{(2k'_{t2})^{0.5}k_t}\left(\frac{k'_{t2}k_p[M]}{(k'_i K'_i X_3[M] - k_d K_d X_1[S])^{0.5}} + \frac{k_{t2c}k'_p(k'_i K'_i X_3[M] - k_d K_d X_1[S])^{0.5}}{k'_i K'_i X_3}\right) \qquad (30)$$

$$R_h = \left(1 - \frac{k_{t2c}k_d K_d X_1[S]}{k_t k'_i K'_i X_3[M]}\right)k'_p[M] \times \frac{(k'_i K'_i X_3[M] - k_d K_d X_1[S])^{0.5}[Ce^{IV}]^{0.5}}{(2k'_{t2})^{0.5}} \qquad (31)$$

$$GE = \left(1 + \frac{k_{t2c}k'_p(k'_i K'_i X_3[M] - k_d K_d X_1[S])}{k'_{t2}k_p k'_i K'_i X_3[M]}\right) \div \left(1 + \frac{k_t k'_p(k'_i K'_i X_3[M] - k_d K_d X_1[S])}{k'_{t2}k_p k_d K_d X_1[S]}\right) \qquad (32)$$

$$R_{Ce} = [Ce^{IV}](k_d K_d X_1[S] + k'_i K'_i X_3[M]) \qquad (24)$$

$$v_g = \frac{1}{(2k'_{t2})^{0.5}k_t[Ce^{IV}]^{0.5}}\left(\frac{k'_{t2}k_p[M]}{(k'_i K'_i X_3[M] - k_d K_d X_1[S])^{0.5}} + \frac{k_t k'_p(k'_i K'_i X_3[M] - k_d K_d X_1[S])^{0.5}}{k'_i K'_i X_3}\right) \qquad (33)$$

$$v_h = \frac{k'_p(k'_i K'_i X_3[M] - k_d K_d X_1[S])^{0.5}}{(2k'_{t2})^{0.5}k'_i K'_i X_3[Ce^{IV}]^{0.5}} \qquad (34)$$

$$N_g = k_d K_d X_1[S][Ce^{IV}] \qquad (35)$$

$$N_h = n\left(1 - \frac{k_{t2c}k_d K_d X_1[S]}{k_t k'_i K'_i X_3[M]}\right)k'_i K'_i X_3[Ce^{IV}][M] \qquad (36)$$

$$GF \propto N_g = k_d K_d X_1[S][Ce^{IV}] \qquad (35)$$

$$BE = 1 \qquad (29)$$

Case (Abβ)

$$R_g = \frac{k_d K_d X_1[S]}{2k_t k'_{t1}K_t X_4}\left(\frac{k_p(k'_{t1}K_t X_4)^2[Ce^{IV}][M]}{k'_i K'_i X_3[M] - k_d K_d X_1[S]} + \frac{2k_{t2c}k'_p(k'_i K'_i X_3[M] - k_d K_d X_1[S])}{k'_i K'_i X_3}\right) \qquad (37)$$

$$R_h = \left(1 - \frac{k_{t2c}k_d K_d X_1[S]}{k_t k'_i K'_i X_3[M]}\right)k'_p[M]\,\frac{k'_i K'_i X_3[M] - k_d K_d X_1[S]}{k'_{t1}K_t X_4} \qquad (38)$$

$$GE = \left(1 + \frac{2k_{t2c}k'_p(k'_i K'_i X_3[M] - k_d K_d X_1[S])^2}{(k'_{t1}K_t X_4)^2 k_p k'_i K'_i X_3[M][Ce^{IV}]}\right) \div \left(1 + \frac{2k_t k'_p(k'_i K'_i X_3[M] - k_d K_d X_1[S])^2}{(k'_{t1}K_t X_4)^2 k_p k_d K_d X_1[S][Ce^{IV}]}\right) \qquad (39)$$

$$R_{Ce} = [Ce^{IV}]\{k_d K_d X_1[S] + (1+x)k'_i K'_i X_3[M]\} \qquad (40)$$

$$v_g = \frac{1}{2k_t k'_{t1}K_t X_4}\left(\frac{k_p(k'_{t1}K_t X_4)^2[M]}{k'_i K'_i X_3[M] - k_d K_d X_1[S]} + \frac{2k_t k'_p(k'_i K'_i X_3[M] - k_d K_d X_1[S])}{k'_i K'_i X_3[Ce^{IV}]}\right) \qquad (41)$$

$$v_{\mathrm{h}}=\frac{k'_{\mathrm{p}}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])}{k'_{\mathrm{t1}}K_{\mathrm{t}}X_4k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]} \tag{42}$$

$$N_{\mathrm{g}}=k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}] \tag{35}$$

$$N_{\mathrm{h}}=\left(1-\frac{k_{\mathrm{t2c}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]}{k_{\mathrm{t}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]}\right)k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}][\mathrm{M}] \tag{36}$$

$$GF\propto N_{\mathrm{g}}=k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}] \tag{35}$$

$$BE=1 \tag{29}$$

Case (Abγ)

$$R_{\mathrm{g}}=\frac{k_{\mathrm{p}}k_{\mathrm{tH}}X_5(k_{\mathrm{d}}K_{\mathrm{d}}X_1)^2[\mathrm{S}]^2[\mathrm{Ce^{IV}}]}{k_{\mathrm{t}}k_{\mathrm{i}}K_{\mathrm{i}}X_2(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])}+\frac{k'_{\mathrm{p}}k_{\mathrm{t2c}}k_{\mathrm{i}}K_{\mathrm{i}}X_2(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])[\mathrm{M}]}{2k_{\mathrm{t}}k_{\mathrm{tH}}X_5k'_{\mathrm{i}}K'_{\mathrm{i}}X_3} \tag{43}$$

$$R_{\mathrm{h}}=\left(1-\frac{k_{\mathrm{t2c}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]}{k_{\mathrm{t}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]}\right)k'_{\mathrm{p}}[\mathrm{M}]^2\times\frac{k_{\mathrm{i}}K_{\mathrm{i}}X_2(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]} \tag{44}$$

$$GE=\left(1+\frac{k_{\mathrm{t2c}}k'_{\mathrm{p}}(k_{\mathrm{i}}K_{\mathrm{i}}X_2)^2[\mathrm{M}](k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])^2}{2(k_{\mathrm{tH}}X_5)^2k_{\mathrm{p}}(k_{\mathrm{d}}K_{\mathrm{d}}X_1)^2k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{S}]^2[\mathrm{Ce^{IV}}]}\right)\div\left(1+\frac{k_{\mathrm{t}}k'_{\mathrm{p}}(k_{\mathrm{i}}K_{\mathrm{i}}X_2[\mathrm{M}])^2(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])^2}{2(k_{\mathrm{tH}}X_5)^2k_{\mathrm{p}}(k_{\mathrm{d}}K_{\mathrm{d}}X_1)^3[\mathrm{S}]^3[\mathrm{Ce^{IV}}]}\right) \tag{45}$$

$$R_{\mathrm{Ce}}=[\mathrm{Ce^{IV}}](k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]) \tag{24}$$

$$v_{\mathrm{g}}=\frac{k_{\mathrm{p}}k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]}{k_{\mathrm{t}}k_{\mathrm{i}}K_{\mathrm{i}}X_2(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])}+\frac{k'_{\mathrm{p}}k_{\mathrm{i}}K_{\mathrm{i}}X_2(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])[\mathrm{M}]}{2k_{\mathrm{tH}}X_5k'_{\mathrm{i}}K'_{\mathrm{i}}X_3k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}]} \tag{46}$$

$$v_{\mathrm{h}}=\frac{k'_{\mathrm{p}}k_{\mathrm{i}}K_{\mathrm{i}}X_2[\mathrm{M}](k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}X_1k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{S}][\mathrm{Ce^{IV}}]} \tag{47}$$

$$N_{\mathrm{g}}=k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}] \tag{35}$$

$$N_{\mathrm{h}}=\left(1-\frac{k_{\mathrm{t2c}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]}{k_{\mathrm{t}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]}\right)k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}][\mathrm{M}] \tag{36}$$

$$GF\propto N_{\mathrm{g}}=k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}] \tag{35}$$

$$BE=1 \tag{29}$$

Case (Ac)

$$R_{\mathrm{g}}=k_{\mathrm{p}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{M}]/k_{\mathrm{t1}}K_{\mathrm{t}}X_4 \tag{48}$$

$$R_{\mathrm{h}}=k'_{\mathrm{p}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]^2/k'_{\mathrm{t1}}K_{\mathrm{t}}X_4 \tag{49}$$

$$GE=k'_{\mathrm{t1}}k_{\mathrm{p}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]/(k'_{\mathrm{t1}}k_{\mathrm{p}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+k_{\mathrm{t1}}k'_{\mathrm{p}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]) \tag{50}$$

$$R_{\mathrm{Ce}}=2[\mathrm{Ce^{IV}}](k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]) \tag{51}$$

$$v_{\mathrm{g}}=k_{\mathrm{p}}[\mathrm{M}]/k_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}] \tag{52}$$

$$v_{\mathrm{h}}=k'_{\mathrm{p}}[\mathrm{M}]/k'_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}] \tag{53}$$

$$N_{\mathrm{g}}=k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}] \tag{35}$$

$$N_{\mathrm{h}}=k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}][\mathrm{M}] \tag{54}$$

$$GF\propto N_{\mathrm{g}}=k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}] \tag{35}$$

$$BE=1 \tag{29}$$

Case (BIa)

$$R_{\mathrm{g}}=k_{\mathrm{p}}[\mathrm{M}]^{1.5}(k_{\mathrm{d}}K_{\mathrm{d}}k_{\mathrm{i}}K_{\mathrm{i}}X_2[\mathrm{S}]/2k_{\mathrm{t2}}k_0K_0)^{0.5} \tag{55}$$

$$R_{\mathrm{h}}=k'_{\mathrm{p}}[\mathrm{M}]^{1.5}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]/2k'_{\mathrm{t2}})^{0.5} \tag{22}$$

$$GE=\frac{k_{\mathrm{p}}(2k'_{\mathrm{t2}}k_{\mathrm{d}}K_{\mathrm{d}}k_{\mathrm{i}}K_{\mathrm{i}}X_2[\mathrm{S}])^{0.5}}{k_{\mathrm{p}}(2k'_{\mathrm{t2}}k_{\mathrm{d}}K_{\mathrm{d}}k_{\mathrm{i}}K_{\mathrm{i}}X_2[\mathrm{S}])^{0.5}+k'_{\mathrm{p}}(2k_{\mathrm{t2}}k_0K_0k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}])^{0.5}} \tag{56}$$

$$R_{\mathrm{Ce}}=[\mathrm{Ce^{IV}}](2k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]) \tag{57}$$

$$v_{\mathrm{g}}=k_{\mathrm{p}}(k_0K_0[\mathrm{M}])^{0.5}/(2k_{\mathrm{t2}}k_{\mathrm{d}}K_{\mathrm{d}}k_{\mathrm{i}}K_{\mathrm{i}}X_2[\mathrm{S}])^{0.5} \tag{58}$$

$$v_{\mathrm{h}}=k'_{\mathrm{p}}[\mathrm{M}]^{0.5}/(2k'_{\mathrm{t2}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}])^{0.5} \tag{26}$$

$$N_{\mathrm{g}}=nk_{\mathrm{d}}K_{\mathrm{d}}k_{\mathrm{i}}K_{\mathrm{i}}X_2[\mathrm{S}][\mathrm{M}]/k_0K_0 \tag{59}$$

$$N_{\mathrm{h}}=nk'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}][\mathrm{M}] \tag{28}$$

$$GF\propto N_{\mathrm{g}}=nk_{\mathrm{d}}K_{\mathrm{d}}k_{\mathrm{i}}K_{\mathrm{i}}X_2[\mathrm{S}][\mathrm{M}]/k_0K_0 \tag{59}$$

$$BE=k_{\mathrm{i}}K_{\mathrm{i}}X_2[\mathrm{M}]/k_0K_0X_1[\mathrm{Ce^{IV}}] \tag{60}$$

Case (BIbα)

$$R_{\mathrm{g}}=\frac{k_{\mathrm{i}}K_{\mathrm{i}}X_2k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}][\mathrm{M}]^{1.5}}{(2k'_{\mathrm{t2}})^{0.5}k_{\mathrm{t}}k_0K_0}\left(\frac{k'_{\mathrm{t2}}k_{\mathrm{p}}}{(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]-k_{\mathrm{i}}K_{\mathrm{i}}X_2k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0)^{0.5}}+\frac{k_{\mathrm{t2c}}k'_{\mathrm{p}}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]-k_{\mathrm{i}}K_{\mathrm{i}}X_2k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0)^{0.5}}{k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]}\right) \tag{61}$$

$$R_{\mathrm{h}}=\left(1-\frac{k_{\mathrm{t2c}}k_{\mathrm{i}}K_{\mathrm{i}}X_2k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]}{k_{\mathrm{t}}k_0K_0k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]}\right)k'_{\mathrm{p}}[\mathrm{M}]^{1.5}\times\frac{(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]-k_{\mathrm{i}}K_{\mathrm{i}}X_2k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0)^{0.5}}{(2k'_{\mathrm{t2}})^{0.5}} \tag{62}$$

$$GE=\left(1+\frac{k_{\mathrm{t2c}}k'_{\mathrm{p}}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]-k_{\mathrm{i}}K_{\mathrm{i}}X_2k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0)}{k'_{\mathrm{t2}}k_{\mathrm{p}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]}\right)\div\left(1+\frac{k_{\mathrm{t}}k'_{\mathrm{p}}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]-k_{\mathrm{i}}K_{\mathrm{i}}X_2k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0)}{k'_{\mathrm{t2}}k_{\mathrm{p}}k_{\mathrm{i}}K_{\mathrm{i}}X_2k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0}\right) \tag{63}$$

$$R_{\mathrm{Ce}}=[\mathrm{Ce^{IV}}](2k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]) \tag{57}$$

$$v_{\mathrm{g}}=\frac{[\mathrm{M}]^{0.5}}{(2k'_{\mathrm{t2}})^{0.5}k_{\mathrm{t}}}\left(\frac{k'_{\mathrm{t2}}k_{\mathrm{p}}}{(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]-k_{\mathrm{i}}K_{\mathrm{i}}X_2k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0)^{0.5}}+\frac{k_{\mathrm{t}}k'_{\mathrm{p}}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]-k_{\mathrm{i}}K_{\mathrm{i}}X_2k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0)^{0.5}}{k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]}\right) \tag{64}$$

$$v_{\mathrm{h}}=\frac{k'_{\mathrm{p}}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]-k_{\mathrm{i}}K_{\mathrm{i}}X_2k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0)^{0.5}[\mathrm{M}]^{0.5}}{(2k'_{\mathrm{t2}})^{0.5}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]} \tag{65}$$

$$N_g = k_i K_i X_2 k_d K_d[\mathrm{S}][\mathrm{M}]/k_0 K_0 \tag{66}$$

$$N_h = n\left(1 - \frac{k_{t2c} k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0}{k_t k_i' K_i' X_3[\mathrm{Ce^{IV}}]}\right) k_i' K_i' X_3[\mathrm{Ce^{IV}}][\mathrm{M}] \tag{67}$$

$$GF \propto N_g = k_i K_i X_2 k_d K_d[\mathrm{S}][\mathrm{M}]/k_0 K_0 \tag{66}$$

$$BE = k_i K_i X_2[\mathrm{M}]/k_0 K_0 X_1[\mathrm{Ce^{IV}}] \tag{60}$$

Case (BIbβ)

$$R_g = \frac{k_i K_i X_2 k_d K_d[\mathrm{S}][\mathrm{M}]}{2k_0 K_0 k_t k_{t1}' K_t X_4} \times \left(\frac{k_p (k_{t1}' K_t X_4)^2 [\mathrm{Ce^{IV}}]}{k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0} + \frac{2k_{t2c} k_p' (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)[\mathrm{M}]}{k_i' K_i' X_3[\mathrm{Ce^{IV}}]^2}\right) \tag{68}$$

$$R_h = \left(1 - \frac{k_{t2c} k_i K_i X_2 k_d K_d[\mathrm{S}]}{k_t k_0 K_0 k_i' K_i' X_3[\mathrm{Ce^{IV}}]}\right) k_p'[\mathrm{M}]^2 \times \frac{k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0}{k_{t1}' K_t X_4[\mathrm{Ce^{IV}}]} \tag{69}$$

$$GE = \left(1 + \frac{2k_{t2c} k_p' (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)^2 [\mathrm{M}]}{(k_{t1}' K_t X_4)^2 k_p k_i' K_i' X_3[\mathrm{Ce^{IV}}]^3}\right) \div \left(1 + \frac{2k_t k_p' (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)^2 [\mathrm{M}]}{(k_{t1}' K_t X_4)^2 k_p k_i K_i X_2 k_d K_d[\mathrm{Ce^{IV}}]^2[\mathrm{S}]/k_0 K_0}\right) \tag{70}$$

$$R_{\mathrm{Ce}} = [\mathrm{Ce^{IV}}]\{2k_d K_d X_1[\mathrm{S}] + (1+x) k_i' K_i' X_3[\mathrm{M}]\} \tag{71}$$

$$v_g = \frac{1}{2k_t k_{t1}' K_t X_4}\left(\frac{k_p (k_{t1}' K_t X_4)^2 [\mathrm{Ce^{IV}}]}{k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0} + \frac{2k_t k_p' (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)[\mathrm{M}]}{k_i' K_i' X_3[\mathrm{Ce^{IV}}]^2}\right) \tag{72}$$

$$v_h = \frac{k_p (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)}{k_{t1}' K_t X_4 k_i' K_i' X_3[\mathrm{Ce^{IV}}]^2} \tag{73}$$

$$N_g = k_i K_i X_2 k_d K_d[\mathrm{S}][\mathrm{M}]/k_0 K_0 \tag{66}$$

$$N_h = n\left(1 - \frac{k_{t2c} k_i K_i X_2 k_d K_d[\mathrm{S}]}{k_t k_0 K_0 k_i' K_i' X_3[\mathrm{Ce^{IV}}]}\right) k_i' K_i' X_3[\mathrm{Ce^{IV}}][\mathrm{M}] \tag{67}$$

$$GF \propto N_g = k_i K_i X_2 k_d K_d[\mathrm{S}][\mathrm{M}]/k_0 K_0 \tag{66}$$

$$BE = k_i K_i X_2[\mathrm{M}]/k_0 K_0 X_1[\mathrm{Ce^{IV}}] \tag{60}$$

Case (BIbγ)

$$R_g = \frac{k_p k_{tH} X_5 k_i K_i X_2 (k_d K_d)^2 [\mathrm{S}]^2 [\mathrm{M}]^2}{k_t (k_0 K_0)^2 (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)} + \frac{k_{t2c} k_p' k_i K_i X_2 (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)[\mathrm{M}]^3}{2k_t k_{tH} X_5} \tag{74}$$

$$R_h = \left(1 - \frac{k_{t2c} k_i K_i X_2 k_d K_d[\mathrm{S}]}{k_t k_0 K_0 k_i' K_i' X_3[\mathrm{Ce^{IV}}]}\right) k_p'[\mathrm{M}]^2 \times \frac{k_0 K_0 (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)}{2k_{tH} X_5 k_d K_d[\mathrm{S}]} \tag{75}$$

$$GE = \left(1 + \frac{k_{t2c} k_p' (k_0 K_0)^2 (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)^2 [\mathrm{M}]}{2(k_{tH} X_5)^2 k_p (k_d K_d)^2 k_i' K_i' X_3[\mathrm{S}]^2 [\mathrm{Ce^{IV}}]}\right) \div \left(1 + \frac{k_t k_p' (k_0 K_0)^3 (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)^2 [\mathrm{M}]}{2(k_{tH} X_5)^2 k_p k_i K_i X_2 (k_d K_d)^3 [\mathrm{S}]^3}\right) \tag{76}$$

$$R_{\mathrm{Ce}} = [\mathrm{Ce^{IV}}](2k_d K_d X_1[\mathrm{S}] + k_i' K_i' X_3[\mathrm{M}]) \tag{57}$$

$$v_g = \frac{k_p k_{tH} X_5 k_d K_d[\mathrm{S}][\mathrm{M}]}{k_t k_0 K_0 (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)} + \frac{k_p' k_i K_i X_2 (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)[\mathrm{M}]^3}{2k_{tH} X_5} \tag{77}$$

$$v_h = \frac{k_p' k_0 K_0 (k_i' K_i' X_3[\mathrm{Ce^{IV}}] - k_i K_i X_2 k_d K_d[\mathrm{S}]/k_0 K_0)[\mathrm{M}]}{2k_{tH} X_5 k_d K_d k_i' K_i' X_3[\mathrm{Ce^{IV}}][\mathrm{S}]} \tag{78}$$

$$N_g = k_i K_i X_2 k_d K_d[\mathrm{S}][\mathrm{M}]/k_0 K_0 \tag{66}$$

$$N_h = \left(1 - \frac{k_{t2c} k_i K_i X_2 k_d K_d[\mathrm{S}]}{k_t k_0 K_0 k_i' K_i' X_3[\mathrm{Ce^{IV}}]}\right) k_i' K_i' X_3[\mathrm{Ce^{IV}}][\mathrm{M}] \tag{67}$$

$$GF \propto N_g = k_i K_i X_2 k_d K_d[\mathrm{S}][\mathrm{M}]/k_0 K_0 \tag{66}$$

$$BE = k_i K_i X_2[\mathrm{M}]/k_0 K_0 X_1[\mathrm{Ce^{IV}}] \tag{60}$$

Case (BIc)

$$R_g = k_p k_d K_d k_i K_i X_2[\mathrm{S}][\mathrm{M}]^2/k_0 K_0 k_{t1} K_t X_4[\mathrm{Ce^{IV}}] \tag{79}$$

$$R_h = k_p' k_i' K_i' X_3[\mathrm{M}]^2/k_{t1}' K_t X_4 \tag{49}$$

$$GE = \frac{k_{t1}' k_p k_d K_d k_i K_i X_2[\mathrm{S}]}{k_{t1}' k_p k_d K_d k_i K_i X_2[\mathrm{S}] + k_{t1} k_p' k_0 K_0 k_i' K_i' X_3[\mathrm{Ce^{IV}}]} \tag{80}$$

$$R_{\mathrm{Ce}} = 2[\mathrm{Ce^{IV}}](k_d K_d X_1[\mathrm{S}] + k_i' K_i' X_3[\mathrm{M}]) \tag{51}$$

$$v_g = k_p[\mathrm{M}]/k_{t1} K_t X_4[\mathrm{Ce^{IV}}] \tag{52}$$

$$v_h = k_p'[\mathrm{M}]/k_{t1}' K_t X_4[\mathrm{Ce^{IV}}] \tag{53}$$

$$N_g = k_d K_d k_i K_i X_2[\mathrm{S}][\mathrm{M}]/k_0 K_0 \tag{66}$$

$$N_h = k_i' K_i' X_3[\mathrm{Ce^{IV}}][\mathrm{M}] \tag{54}$$

$$GF \propto N_g = k_d K_d k_i K_i X_2[\mathrm{S}][\mathrm{M}]/k_0 K_0 \tag{66}$$

$$BE = k_i K_i X_2[\mathrm{M}]/k_0 K_0 X_1[\mathrm{Ce^{IV}}] \tag{60}$$

Case (Yd)

$$R_g = k_p k_i K_i X_2[\mathrm{M}]^2/2k_{tR} X_6 \tag{81}$$

$$R_h = k_p'[\mathrm{M}]^{1.5}(k_i' K_i' X_3[\mathrm{Ce^{IV}}]/2k_{t2}')^{0.5} \tag{22}$$

$$GE = \frac{k_p k_i K_i X_2[\mathrm{M}]^2}{k_p k_i K_i X_2[\mathrm{M}]^2 + 2k_{tR} X_6 k_p'[\mathrm{M}]^{1.5}(k_i' K_i' X_3[\mathrm{Ce^{IV}}]/2k_{t2}')^{0.5}} \tag{82}$$

$$R_{\mathrm{Ce}}=[\mathrm{Ce^{IV}}](k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]) \tag{24}$$

$$v_{\mathrm{g}}=k_{\mathrm{p}}k_{\mathrm{i}}K_{\mathrm{i}}X_2[\mathrm{M}]^2/k_{\mathrm{tR}}X_6k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}] \tag{83}$$

$$v_{\mathrm{h}}=k'_{\mathrm{p}}[\mathrm{M}]^{0.5}/(2k'_{\mathrm{t2}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}])^{0.5} \tag{26}$$

$$N_{\mathrm{g}}=0.5k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}] \tag{84}$$

$$N_{\mathrm{h}}=k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}][\mathrm{Ce^{IV}}] \tag{56}$$

$$GF\propto 2N_{\mathrm{g}}=k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}] \tag{85}$$

$$BE=0.5 \tag{86}$$

Case (BIIα′)

$$R_{\mathrm{g}}=\frac{k'_{\mathrm{p}}k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3)^{0.5}[\mathrm{S}][\mathrm{M}]^{1.5}[\mathrm{Ce^{IV}}]^{0.5}}{(2k'_{\mathrm{t2}})^{0.5}k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]+k'_{\mathrm{t2}}k_0K_0(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}][\mathrm{Ce^{IV}}])^{0.5}} \tag{87}$$

$$R_{\mathrm{h}}=\left(1-\frac{2k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]+k_0K_0(2k'_{\mathrm{t2}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}][\mathrm{Ce^{IV}}])^{0.5}}\right)\times\frac{k'_{\mathrm{p}}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3)^{0.5}[\mathrm{M}]^{1.5}[\mathrm{Ce^{IV}}]^{0.5}}{(2k'_{\mathrm{t2}})^{0.5}} \tag{88}$$

$$GE=\frac{2k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]+k_0K_0(2k'_{\mathrm{t2}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}][\mathrm{Ce^{IV}}])^{0.5}} \tag{89}$$

$$R_{\mathrm{Ce}}=[\mathrm{Ce^{IV}}](2k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]) \tag{57}$$

$$v_{\mathrm{g}}=k'_{\mathrm{p}}[\mathrm{M}]^{0.5}/(2k'_{\mathrm{t2}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}])^{0.5} \tag{90}$$

$$v_{\mathrm{h}}=v_{\mathrm{g}}=k'_{\mathrm{p}}[\mathrm{M}]^{0.5}/(2k'_{\mathrm{t2}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}])^{0.5} \tag{90}$$

$$N_{\mathrm{g}}=\frac{2k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{S}][\mathrm{M}][\mathrm{Ce^{IV}}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]+k_0K_0(2k'_{\mathrm{t2}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}][\mathrm{Ce^{IV}}])^{0.5}} \tag{91}$$

$$N_{\mathrm{h}}=n\left(1-\frac{2k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]+k_0K_0(2k'_{\mathrm{t2}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}][\mathrm{Ce^{IV}}])^{0.5}}\right)\times k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}][\mathrm{M}] \tag{92}$$

$$GF\propto N_{\mathrm{g}}=\frac{2k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{S}][\mathrm{M}][\mathrm{Ce^{IV}}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]+k_0K_0(2k'_{\mathrm{t2}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}][\mathrm{Ce^{IV}}])^{0.5}} \tag{91}$$

$$BE=\frac{2k_{\mathrm{tHc}}X_5k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+k_0K_0X_1(2k'_{\mathrm{t2}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}][\mathrm{Ce^{IV}}])^{0.5}} \tag{93}$$

Case (BIIβ′)

$$R_{\mathrm{g}}=\frac{2k'_{\mathrm{p}}k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}k_0K_0k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{S}][\mathrm{M}]^2[\mathrm{Ce^{IV}}]}{(2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]+k_0K_0k'_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}])^2} \tag{94}$$

$$R_{\mathrm{h}}=\left(1-\frac{2k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0+k'_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}]}\right)\times\frac{k'_{\mathrm{p}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]^2[\mathrm{Ce^{IV}}]}{2k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0+k'_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}]} \tag{95}$$

$$GE=\frac{2k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]+k_0K_0k'_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}]} \tag{96}$$

$$R_{\mathrm{Ce}}=[\mathrm{Ce^{IV}}]\{2k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+(1+x)k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]\} \tag{71}$$

$$v_{\mathrm{g}}=\frac{k'_{\mathrm{p}}[\mathrm{M}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0+k'_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}]} \tag{97}$$

$$v_{\mathrm{h}}=v_{\mathrm{g}}=\frac{k'_{\mathrm{p}}[\mathrm{M}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0+k'_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}]} \tag{97}$$

$$N_{\mathrm{g}}=\frac{2k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{S}][\mathrm{M}][\mathrm{Ce^{IV}}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]+k_0K_0k'_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}]} \tag{98}$$

$$N_{\mathrm{h}}=\left(1-\frac{2k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]/k_0K_0+k'_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}]}\right)\times k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}][\mathrm{M}] \tag{99}$$

$$GF\propto N_{\mathrm{g}}=\frac{2k_{\mathrm{tHc}}X_5k_{\mathrm{d}}K_{\mathrm{d}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{S}][\mathrm{M}][\mathrm{Ce^{IV}}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}[\mathrm{S}]+k_0K_0k'_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}]} \tag{98}$$

$$BE=\frac{2k_{\mathrm{tHc}}X_5k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]}{2k_{\mathrm{tH}}X_5k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+k_0K_0X_1k'_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce^{IV}}]} \tag{100}$$

Case (Cα′)

$$R_{\mathrm{g}}=\frac{k'_{\mathrm{p}}k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}]^{0.5}}{(2k'_{\mathrm{t2}})^{0.5}k_{\mathrm{tH}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])^{0.5} \tag{101}$$

$$R_{\mathrm{h}}=\left(1-\frac{k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]}{k_{\mathrm{tH}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]}\right)k'_{\mathrm{p}}[\mathrm{M}]\times\frac{(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])^{0.5}[\mathrm{Ce^{IV}}]^{0.5}}{(2k'_{\mathrm{t2}})^{0.5}} \tag{102}$$

$$GE=k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]/k_{\mathrm{tH}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}] \tag{103}$$

$$R_{\mathrm{Ce}}=[\mathrm{Ce^{IV}}](k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]) \tag{24}$$

$$v_{\mathrm{g}}=\frac{k'_{\mathrm{p}}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])^{0.5}}{(2k'_{\mathrm{t2}})^{0.5}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]^{0.5}} \tag{34}$$

$$v_{\mathrm{h}}=v_{\mathrm{g}}=\frac{k'_{\mathrm{p}}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])^{0.5}}{(2k'_{\mathrm{t2}})^{0.5}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}]^{0.5}} \tag{34}$$

$$N_{\mathrm{g}}=k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}]/k_{\mathrm{tH}} \tag{104}$$

$$N_{\mathrm{h}}=n\left(1-\frac{k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]}{k_{\mathrm{tH}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]}\right)k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{Ce^{IV}}][\mathrm{M}] \tag{105}$$

$$GF\propto N_{\mathrm{g}}=k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce^{IV}}]/k_{\mathrm{tH}} \tag{104}$$

$$BE=k_{\mathrm{tHc}}/k_{\mathrm{tH}} \tag{106}$$

Case (Cβ′)

$$R_{\mathrm{g}}=\frac{k'_{\mathrm{p}}k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]}{k_{\mathrm{tH}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3k'_{\mathrm{t1}}K_{\mathrm{t}}X_4}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]) \tag{107}$$

$$R_{\mathrm{h}}=\left(1-\frac{k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]}{k_{\mathrm{tH}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]}\right)\frac{k'_{\mathrm{p}}(k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])[\mathrm{M}]}{k'_{\mathrm{t1}}K_{\mathrm{t}}X_4} \tag{108}$$

$$GE=k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]/k_{\mathrm{tH}}k'_{\mathrm{i}}K'_{\mathrm{i}}X_3[\mathrm{M}] \tag{103}$$

$$R_{\mathrm{Ce}}=[\mathrm{Ce}^{\mathrm{IV}}]\{k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]+(1+x)k_{\mathrm{i}}'K_{\mathrm{i}}'X_3[\mathrm{M}]\} \tag{40}$$

$$v_{\mathrm{g}}=\frac{k_{\mathrm{p}}'(k_{\mathrm{i}}'K_{\mathrm{i}}'X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])}{k_{\mathrm{t1}}'K_{\mathrm{t}}X_4k_{\mathrm{i}}'K_{\mathrm{i}}'X_3[\mathrm{Ce}^{\mathrm{IV}}]} \tag{109}$$

$$v_{\mathrm{h}}=v_{\mathrm{g}}=\frac{k_{\mathrm{p}}'(k_{\mathrm{i}}'K_{\mathrm{i}}'X_3[\mathrm{M}]-k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])}{k_{\mathrm{t1}}'K_{\mathrm{t}}X_4k_{\mathrm{i}}'K_{\mathrm{i}}'X_3[\mathrm{Ce}^{\mathrm{IV}}]} \tag{109}$$

$$N_{\mathrm{g}}=k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce}^{\mathrm{IV}}]/k_{\mathrm{tH}} \tag{104}$$

$$N_{\mathrm{h}}=\left(1-\frac{k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}]}{k_{\mathrm{tH}}k_{\mathrm{i}}'K_{\mathrm{i}}'X_3[\mathrm{M}]}\right)k_{\mathrm{i}}'K_{\mathrm{i}}'X_3[\mathrm{Ce}^{\mathrm{IV}}][\mathrm{M}] \tag{110}$$

$$GF\propto N_{\mathrm{g}}=k_{\mathrm{tHc}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}][\mathrm{Ce}^{\mathrm{IV}}]/k_{\mathrm{tH}} \tag{104}$$

$$BE=k_{\mathrm{tHc}}/k_{\mathrm{tH}} \tag{106}$$

In equations (40) and (71), x is the fraction of homopolymer radical chains terminated by ceric ions. In equations (27), (28), (36), (59), (67), (92) and (105), n is a proportionality coefficient that takes values in the range 0.5–1. The value 0.5 corresponds to termination of polymer chains by pure combination and the value 1 corresponds to pure disproportionation.

A correlation is established between cases and grafting mechanisms as follows: the mechanism *from* occurs when the cases (Aa), (Ac), (BIa), (BIc) and (Yd) are applicable; the mechanism *from-to* occurs in the cases (Abα), (Abβ), (Abγ), (BIbα), (BIbβ) and (BIbγ); and the mechanism *onto* occurs in the cases (BIIα'), (BIIβ'), (Cα') and (Cβ').

Some structural knowledge of the copolymer can be deduced from the correlation of cases and grafting mechanisms. The mechanism *from* can lead to branches (cases (Aa), (Ac), (BIa), (BIc) and (Yd)), loops (cases (Aa), (BIa) and (Yd)) and crosslinks (cases (Aa), (BIa) and (Yd)) of synthetic polymer attached to cellulose. The mechanism *from-to* can only lead to branches (cases (Abα), (Abβ), (Abγ), (BIbα), (BIbβ) and (BIbγ)) and such is also the case with the mechanism *onto* (cases (BIIα'), (BIIβ'), (Cα') and (Cβ')).

In the cases where the approximations (BII) or (C) are made, namely, where the mechanism *onto* occurs, the molecular weights of grafts and homopolymer must be the same if channels and pores of fibrous cellulose allow the accessibility of all sizes of homopolymer radicals.

Applicability of mechanistic cases

Taking into account a certain polymerizing system and particular values of the reaction variables, a particular mechanistic case should be applicable, and the parameter equations derived for such a case in the previous section should account for the results obtained. A sufficiently low variation in any of the reaction variables does not change the applicability of the case; and, on the contrary, a sufficiently high variation leads to the applicability of another case. The latter case should present the same characteristics as the former case except for one. That is, only *one* of the approximations made to develop both cases should be different. For instance, the case (Aa) can undergo direct transition of applicability to the cases (Ac) or (BIa) but not to the case (BIc). The nature of the approximation that undergoes loss of predominance is closely related to the reaction variable that varies. In short, appropriate variations in reaction variables lead to transitions in the applicability of cases. The variation range of a reaction variable needed to pass through the application of a case to reach another one may be big or small.

Variations in reaction variables can be made in experiments to very short reaction times by varying only one of the reaction variables from one test to another in a series. In each one of these tests a mechanistic case should be applicable and account for the results. On the other hand, when grafting polymerization experiments are made to longer reaction times, the effect of reaction conversion must be taken into account. As reaction conversion increases, the reactant concentrations decrease, and they may do so at different rates. If the reaction extent along a test is high enough, its effect can be a successive application of several cases that differ in just one approximation between neighbours.

The transition of applicability between two particular cases is defined by a change in one of the approximations made to derive each case. As any approximation results from the predominance of one competitive reaction over others, the transition between two cases is determined by the ratio of the two rates that define both approximations, i.e. the ratio of predominant rates. And each rate depends on rate constants, accessibilities between reactive species and reactant concentrations. As an explanatory example, the transition between the cases (Aa) and (Ac) is determined by the following ratio of predominant rates:

$$R_{\mathrm{t1}}/R_{\mathrm{t2}}=k_{\mathrm{t1}}K_{\mathrm{t}}X_4[\mathrm{Ce}]^{0.5}/(2k_{\mathrm{t2}}k_{\mathrm{d}}K_{\mathrm{d}}X_1[\mathrm{S}])^{0.5}$$

where R_{t2} is predominant in approximation (a) of case (Aa) and R_{t1} is predominant in approximation (c) of case (Ac). This means that the transition of applicability of (Aa) to (Ac) is attainable, if the ratio $k_{\mathrm{t1}}K_{\mathrm{t}}/k_{\mathrm{t2}}k_{\mathrm{d}}K_{\mathrm{d}}$ is not close to zero, by sufficiently (i) increasing $[\mathrm{Ce}^{\mathrm{IV}}]$, (ii) decreasing [S], (iii) increasing X_4 (the accessibility of $\mathrm{Ce}^{\mathrm{IV}}$ and $\mathrm{S{-}M}_n^{\bullet}$) or (iv) decreasing X_1 (the accessibility of $\mathrm{Ce}^{\mathrm{IV}}$ and S), but it is unaffected by changing [M]. Analogously, the transition of applicability of (Ac) to (Aa) is attainable, if the ratio $k_{t2}k_{\mathrm{d}}K_{\mathrm{d}}/k_{t1}K_t$ is not close to zero, by sufficiently (i) increasing [S], (ii) decreasing $[\mathrm{Ce}^{\mathrm{IV}}]$, (iii) increasing X_1 or (iv) decreasing X_4, but it is unaffected by changing [M]. The effect of changing $[\mathrm{H}^+]$ can be incorporated into the effect of changing $[\mathrm{Ce}^{\mathrm{IV}}]$, as discussed previously^1.

Consequently, a transition of applicability between any two differing cases in one approximation does not occur when the ratio of predominant rates does not depend on the reaction variable being varied or it is not varied enough and the ratio of reactivities or accessibilities in the ratio of predominant rates is close to zero. Therefore, the sequential applicability of cases, when a reaction variable is varied (*Figures 1* to *6*), is destroyed if any of these conditions is not accomplished.

Figures 1 to *6* are deduced by calculating the ratios of predominant rates of any pair of cases. These ratios allow the mechanistic cases to be arranged according to their successive applicability when a reaction variable varies.

As can be seen in these figures, several ways allow drift between cases when a reaction variable varies. A particular polymerizing system will follow one or other of these ways according to the values of reactivities, accessibilities and reactant concentrations in the ratios of predominant rates.

From the relations made above between grafting mechanisms and the electronic nature of vinyl and acrylic monomers, and between grafting mechanisms and cases, a relation between cases and monomers emerges:

electron-poor olefins demand preferably the application of cases derived from the approximations (A) and (BI) and electron-rich olefins require approximations (BII) and (C). This consideration is used in the following as a criterion to approach the transitions between cases when a reaction variable is varied.

The applicability of the cases derived by using the approximation (b) can be of prime importance in chemical systems where the graft polymerization and homopolymerization sites are identical. On the contrary, the applicability of these cases could be negligible where both polymerization sites are different. Some evidence on the nature of the copolymerization and homopolymerization sites (differentiated or identical) can be taken by studying the graft polymerization of monomer mixtures[2].

The mechanistic cases may differ in the relative compatibility between the approximations used in their definition and because of that they may differ in their ranges of applicability when a reaction variable is varied. For instance, the case (Abβ) shows a low internal compatibility in systems where the accessibilities of the reactive species are not restricted, as radicals $S\text{–}M_n^{\bullet}$ terminate *mutually* (with radicals $M_n^{\bullet}$) while a portion of radicals $M_n^{\bullet}$ terminates *linearly* (with ceric ions).

The goal now is to establish sequential applicabilities of cases with prime validity either for electron-poor olefins or for electron-rich olefins, when each one of the reactant concentrations is varied. The transitions of applicability between neighbouring cases are indicated by arrows in *Figures 1* to *6*. An arrow (→) points at a case resulting from a transition from a starting case by increasing the reactant concentration concerned. A double arrow (⇌) means that no transition occurs between the two cases by any change in the reactant concentration concerned. Needless to say, the reverse transitions, occurring by appropriately decreasing reactant concentrations, are not written.

Increasing [M]

(i) Electron-poor olefins. *Figure 1* shows the multi-sequential applicability of mechanistic cases primarily valid for an electron-poor olefin when [M] is increased.

In this arrangement there are two cases (cases (Yd) and (BIc)) that move to other cases when [M] increases because they cannot undergo a transition between them. On the other hand, there is only one case (case (Abα)) to which all other cases tend. This means that case (Yd) or (BIc) is applicable at sufficiently low [M] and the case (Abα) at sufficiently high [M]. The case (Yd) is favoured at 'high' values of the ratio [S]/[Ce^{IV}] and the case (BIc) is favoured at 'low' values of this ratio, as indicated by the ratio of predominant rates of both cases $R_{\mathrm{t1}}/R_{\mathrm{tR}}$.

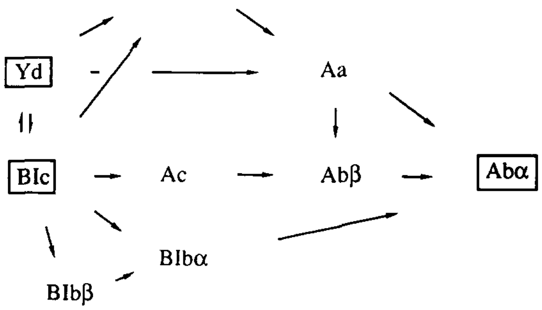

Figure 1 Applicability of mechanistic cases and their transitions of applicability on increasing [M] for electron-poor olefins

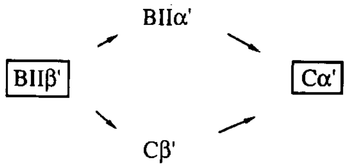

Figure 2 Applicability of mechanistic cases and their transitions of applicability on increasing [M] for electron-rich olefins

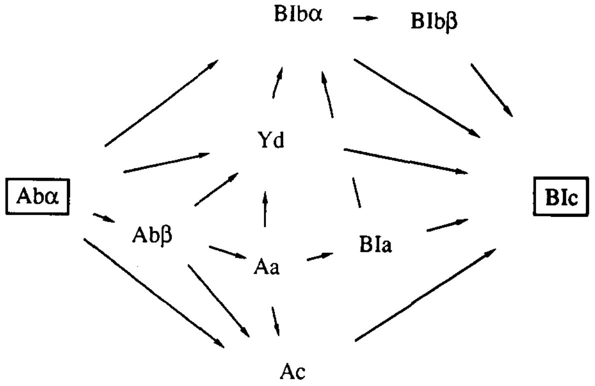

Figure 3 Applicability of mechanistic cases and their transitions of applicability on increasing [Ce^{IV}] for electron-poor olefins

A description of graft polymer formation according to *Figure 1* follows. At sufficiently low [M], cellulose is 'highly' oxidized by Ce^{IV} and a comparatively low fraction of radicals $S^{\bullet}$ add monomer giving rise to radicals $S\text{–}M_n^{\bullet}$ that terminate either with ceric ions (case (BIc)) or with cellulosic radicals (case (Yd)). By increasing [M], the portion of cellulosic radicals trapped by monomer increases (R_i increases) till oxidized cellulose is negligible. From this point onward, R_i remains constant ($R_i = R_d$) though R_i' goes on increasing in parallel to [M] and, consequently, the formation of radicals $M_n^{\bullet}$ grows, favouring the mutual termination of $S\text{–}M_n^{\bullet}$ with $M_n^{\bullet}$ (case (Abα)).

(ii) Electron-rich olefins. *Figure 2* shows the successive applicability of mechanistic cases primarily valid for an electron-rich olefin when [M] is increased.

By increasing [M], the case (BIIβ') can drift to other cases and the case (Cα') is the only end of these transitions. Assuming a sufficiently low monomer concentration, most of the radicals $S^{\bullet}$ formed are oxidized by ceric ions and a minor portion leads to graft polymer by means of a combination reaction with radicals $M_n^{\bullet}$ (case BIIβ')). A large enough increase in the monomer concentration increases the radical $M_n^{\bullet}$ formation (R_i' increases), allowing most of the radicals $S^{\bullet}$ to terminate mutually with radicals $M_n^{\bullet}$ (case (Cα')).

Increasing [Ce^{IV}]

(i) Electron-poor olefins. *Figure 3* shows the multi-sequential applicability of mechanistic cases primarily valid for an electron-poor olefin when [Ce^{IV}] (or [H^+], as previously discussed[1]) is increased.

By sufficiently increasing [Ce^{IV}], the case (Abα) drifts through several cases to reach finally applicability of the case (BIc). When ceric ions are present in a sufficiently low concentration, most of the radicals $S^{\bullet}$ formed add

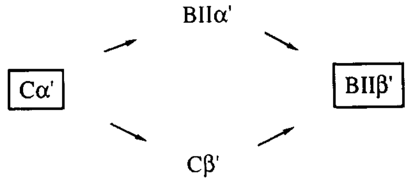

Figure 4 Applicability of mechanistic cases and their transitions of applicability on increasing $[\mathrm{Ce^{IV}}]$ for electron-rich olefins

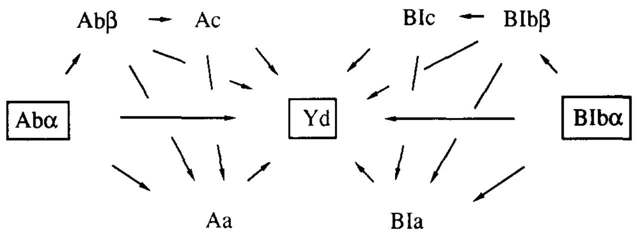

Figure 5 Applicability of mechanistic cases and their transitions of applicability on increasing [S] for electron-poor olefins

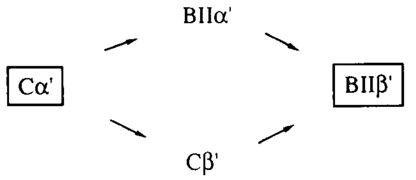

Figure 6 Applicability of mechanistic cases and their transitions of applicability on increasing [S] for electron-rich olefins

monomer and give rise to radicals $\mathrm{S{-}M}_n^\bullet$ terminating mutually with radicals $\mathrm{M}_n^\bullet$ (case (Abα)). By increasing $[\mathrm{Ce^{IV}}]$, several reactions in which ceric ions intervene are favoured. At high enough $[\mathrm{Ce^{IV}}]$, most of the radicals $\mathrm{S}^\bullet$ formed are oxidized by ceric ions; however, a minor portion of radicals $\mathrm{S}^\bullet$, that can be important, adds monomer leading to radicals $\mathrm{S{-}M}_n^\bullet$ that constitute graft polymer after termination by ceric ions (case (BIc)).

(ii) Electron-rich olefins. *Figure 4* shows the successive applicability of mechanistic cases primarily valid for an electron-rich olefin when $[\mathrm{Ce^{IV}}]$ (or $[\mathrm{H^+}]$, as previously discussed[1]) is increased.

Assuming a sufficiently low $[\mathrm{Ce^{IV}}]$, most of the radicals $\mathrm{S}^\bullet$ formed terminate mutually with radicals $\mathrm{M}_n^\bullet$ (case (Cα')) to yield graft polymer. An increase in the ceric-ion concentration favours both the radicals $\mathrm{S}^\bullet$ formation and their oxidation. After sufficiently increasing $[\mathrm{Ce^{IV}}]$, most of the radicals $\mathrm{S}^\bullet$ are oxidized and a comparatively lower amount, that can be important, terminate mutually with radicals $\mathrm{M}_n^\bullet$ (case (BIIβ')).

Increasing [S]

(i) Electron-poor olefins. *Figure 5* shows the multisequential applicability of mechanistic cases primarily valid for an electron-poor olefin when [S] is increased.

As cases derived from approximations (A) and (BI) do not undergo transition between them, the cases (Abα) and (BIbα) can be applicable at low enough cellulose concentration. These cases move independently by sufficiently increasing [S] to reach the case (Yd) as the only goal. Applicability of the case (Abα) is favoured at 'high' values of the ratio $[\mathrm{M}]/[\mathrm{Ce^{IV}}]$ and the case (BIbα) is favoured at 'low' values of this ratio, as indicated by the predominant rates R_i/R_0 of both cases.

At sufficiently low cellulose concentration, most of the radicals $\mathrm{S}^\bullet$ formed add monomer (case (Abα)) or they are oxidized (case (BIbα)); anyway, the radicals $\mathrm{S{-}M}_n^\bullet$ formed terminate mutually with radicals $\mathrm{M}_n^\bullet$ yielding graft polymer. By sufficiently increasing [S], a larger portion of ceric ions is demanded to react with cellulose, decreasing the rates of other reactions of ceric ions and yielding an increased amount of radicals $\mathrm{S}^\bullet$ that favours their mutual termination with radicals $\mathrm{S{-}M}_n^\bullet$ (case (Yd)) to yield graft polymer.

(ii) Electron-rich olefins. *Figure 6* shows the successive applicability of mechanistic cases primarily valid for an electron-rich olefin when [S] is increased.

Assuming a sufficiently low [S], most of the radicals $\mathrm{S}^\bullet$ formed terminate mutually with radicals $\mathrm{M}_n^\bullet$ (case (Cα')) to yield graft polymer. An increase in the cellulose concentration favours the radicals $\mathrm{S}^\bullet$ formation by demanding more ceric ions and, as a consequence, a decrease in the radicals $\mathrm{M}_n^\bullet$ formation (R_i' decreases) occurs. At high enough [S], most of the radicals $\mathrm{S}^\bullet$ are oxidized and a comparatively lower amount, that can be important, terminates mutually with radicals $\mathrm{M}_n^\bullet$ (case BIIβ')).

The sequential applicability of cases shown in *Figures 1* to *6* and the expressions of reaction parameters (equations (21) to (110)) allow one to ascertain the behaviour of reaction parameters when a reaction variable varies. A reaction parameter may show a low sensitivity to the diversity of ways by which the applicability of cases drifts when a reaction variable varies, but the copolymer structure shows a strong dependence on such ways.

Expressions for the reaction variables valid at the reaction transitions (i.e. transitions of applicability of cases) between neighbouring cases in *Figures 1* to *6* can be developed by equating to unity the corresponding ratio of predominant rates[1].

Estimation of values of the reaction variables at which the transitions occur can be made by plotting appropriate equations of reaction parameters in a suitable manner in order to obtain straight lines[1].

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